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# Peer Prediction with Heterogeneous Tasks

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## Abstract

Peer prediction is a method to promote contributions of information by users in settings in which there is no way to verify the quality of responses. In multi-task peer prediction, the reports from users across multiple tasks are used to score contributions. This paper extends the *correlated agreement (CA)* multi-task peer prediction mechanism (Shnayder et al. [2016]) to allow the reports from users to be on heterogeneous tasks, each associated with different distributions on responses. The motivation comes from wanting to elicit user-generated content about places in a city, where tasks vary because places, and questions about places, vary. We prove that the generalized CA mechanism is *informed truthful* under weak conditions, meaning that it is strictly beneficial for a user to invest effort and acquire information, and that truthful reporting is the best strategy when investing effort, as well as an equilibrium. We demonstrate that the mechanism has good incentive properties when tested in simulation on distributions derived from user reports on Google Local Guides.

## 1 Introduction

Peer prediction refers to the problem of scoring information reports in settings where the correctness of a report cannot be verified, either because there is no objectively correct answer or because this answer is too costly to acquire. This problem arises in diverse contexts; e.g., peer assessment of assignments in massive open online courses, and when collecting feedback about a new restaurant. Peer prediction algorithms use reports from multiple participants to score contributions.

Simple approaches compare the responses of two users and award them if they agree. But this does not promote truthful reporting when one user believes that it is unlikely that another user will have the same opinion. This problem can be alleviated by adjusting scores according to the frequency of reports [Jurca and Faltings, 2008, Witkowski and Parkes, 2012, Kamble et al., 2015]. Peer prediction methods can also be made robust to coordinated misreports (see Dasgupta and Ghosh [2013], Shnayder et al. [2016] and Kong and Schoenebeck [2016]).

A limitation of current approaches, however, is that tasks are assumed to be *ex ante* identical, with each task associated with the same distribution on reports. But tasks on platforms such as *Google Local Guides*, which seek to elicit content from users about places in a city, are quite heterogeneous. On this kind of platform, a user is encouraged to answer several different types of questions (= tasks) related to the same place; e.g., “is the restaurant noisy?”, “is it accessible by wheelchair?”, or “does it serve wine?”. Even though the questions are related to the same place, the prior beliefs about the distribution on reports for each type of question may be very different.

We design a new, multi-task peer prediction mechanism (the *correlated agreement-heterogeneous mechanism*) that is responsive to this challenge. This new mechanism shares similar properties with the earlier *correlated agreement* (CA) mechanism [Shnayder et al., 2016]. In particular, it is *informed truthful* under weak conditions, meaning that it is strictly beneficial for a user to invest effort and acquire information, and that truthful reporting is the best strategy when investing effort, as well as an equilibrium. We demonstrate that the mechanism has good incentive properties when tested in simulation on distributions derived from user reports on Google Local Guides.

## 1.1 Related Work

We focus in this brief discussion on mechanisms that are *minimal*, in the sense that they only require signal (or information) reports and do not require belief reports. Miller et al. [2005] introduced the peer prediction problem and proposed a minimal mechanism that has truthful reporting in an equilibrium, however the mechanism’s design requires knowledge of the joint signal distribution and is vulnerable to coordinated misreports.

There followed a large literature on making peer prediction methods more robust, while reducing the amount of knowledge assumed on the part of the designer. For example, one can use marginal probabilities in place of the joint distribution [Witkowski and Parkes, 2012, Jurca and Faltings, 2008] and by coupling the estimation of statistics of the distribution on signals with the scoring algorithm, it is possible to achieve robustness to coordinated misreports [Kamble et al., 2015, Radanovic et al., 2016]. Robustness to coordinated misreports can also be achieved by using reports across multiple tasks along with access to partial information about the joint distribution [Dasgupta and Ghosh, 2013]. See also Kong and Schoenebeck [2016], who provide a theoretical framework for the design of mechanisms that are strongly robust to coordinated misreports.

Shnayder et al. [2016] generalize the mechanism of Dasgupta and Ghosh [2013] to handle multiple signals, and show how the required knowledge about the distribution (the correlation structure on pairs of signals) can be estimated from reports without compromising incentives. Their CA mechanism rewards pairs of reports on the same task (and penalizes pairs of reports on different tasks) based on whether the signals are positively or negatively correlated. To the best of our knowledge, there is no prior work on extending the design of these multiple-task mechanisms to heterogeneous tasks, where pairs of reports may be on different types of tasks, with each task associated with a different signal distribution. This is the focus of the present paper.

## 2 Heterogeneous, Multi-Task Peer Prediction

Consider two agents, 1 and 2, who are members of a large population. Each agent is assigned to a set of  $M = \{1, 2, \dots, m\}$  tasks. We adopt a binary effort model: if an agent invests effort he incurs a cost and obtains an informed *signal*, otherwise the agent receives no signal. There are  $n$  signals. We do not assume that tasks are *ex ante* identical, however, we do assume that the signals for different tasks are drawn independently. Let  $S_k^1$  and  $S_k^2$  respectively be the signals of agents 1 and 2 for task  $k$  (if investing effort). Let  $P_k(i, j) = \Pr[S_k^1 = i, S_k^2 = j]$  be the joint probability for a pair of signals  $(i, j)$  on task  $k$  and let  $P_k(i)$  and  $P_k(j)$  be the corresponding marginal probabilities. We assume that the agents are exchangeable in their roles in these distributions, with the same marginal distributions and joint distributions for any pair of agents.

An agent’s strategy maps every task and every received signal to a reported signal. Agents make reports without knowledge of each others’ reports. We assume that the type of task, and signal about a task (upon investing effort), is the only information available to an agent. For simplicity, we assume that an agent adopts the same strategy across all tasks, and leave the analysis of heterogeneous strategies to future work.<sup>1</sup> We allow the strategy to be randomized, i.e. a probability distribution over the set of possible signals. We will write  $F$  and  $G$  to denote the mixed strategies of agents 1 and 2 respectively. Let  $\mathbb{I}$  denote the truthful strategy i.e.  $\mathbb{I}(j) = j$ . As in Shnayder et al. [2016], we are interested in the following two incentive properties:

**Definition 2.1.** (Strong Truthful) A peer prediction mechanism is *strong truthful* iff for all strategies  $F, G$  we have  $E(\mathbb{I}, \mathbb{I}) \geq E(F, G)$ , and equality may hold only when  $F$  and  $G$  are both the same permutation strategy (i.e. a bijection from received signals to reported signals.)

<sup>1</sup>This is without loss of generality in the homogeneous task setting of Shnayder et al. [2016], but need not be in the present context.

**Definition 2.2.** (Informed Truthful) A peer prediction mechanism is *informed truthful* iff for all strategies  $F, G$  we have  $E(\mathbb{I}, \mathbb{I}) \geq E(F, G)$  and equality may hold only when  $F$  and  $G$  are informed strategies (i.e. reports depend on an agent's signal).

These two truthfulness properties imply that truthful reporting is a strict and weak correlated equilibrium, respectively [Shnayder et al., 2016]. They also ensure that there are no useful, coordinated misreports available to agents.

## 2.1 Delta Matrices

Following [Shnayder et al., 2016], define the following  $n \times n$  matrix for task  $k$ , called the *delta matrix*:

$$\Delta_k(i, j) = P_k(i, j) - P_k(i)P_k(j). \quad (1)$$

Let  $S_k$  be the *sign matrix* of  $\Delta_k$  i.e.  $S_k(i, j) = 1$  if  $\Delta_k(i, j) > 0$  and  $S_k(i, j) = 0$  otherwise. The CA mechanism [Shnayder et al., 2016] supposes that each task  $k$  is *ex ante* identical, and thus has the same delta matrix. Denote this matrix  $\Delta$ , with  $S$  the corresponding sign matrix. The CA mechanism works as follows :

1. Let  $r_k^1$  ( $r_k^2$ ) be the signal reported by agent 1 (2) on task  $k$ .
2. Pick a task  $b$  uniformly at random as the *bonus task*, and pick *penalty tasks*  $l'$  and  $l''$  (with  $l' \neq l''$ ) uniformly at random from the remaining tasks.
3. Pay each agent  $S(r_b^1, r_b^2) - S(r_{l'}^1, r_{l''}^2)$ .

A simple generalization that pays  $S_b(r_b^1, r_b^2) - S_b(r_{l'}^1, r_{l''}^2)$ , where  $S_b$  is the sign matrix corresponding to the bonus task, is not informed truthful for heterogeneous tasks.

**Example 1** (CA is not informed truthful with heterogeneous tasks). Consider three tasks (1, 2 and 3) with the following joint probability distributions

$$P_1 : \begin{bmatrix} Y & N \\ Y & 0.4 & 0.22 \\ N & 0.22 & 0.16 \end{bmatrix} \quad P_2 : \begin{bmatrix} Y & N \\ Y & 0.7 & 0.14 \\ N & 0.14 & 0.02 \end{bmatrix} \quad P_3 : \begin{bmatrix} Y & N \\ Y & 0.4 & 0.22 \\ N & 0.22 & 0.16 \end{bmatrix} \quad (2)$$

and the following sign matrices:

$$\text{sign}(\Delta_1) : \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{sign}(\Delta_2) : \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{sign}(\Delta_3) : \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (3)$$

Suppose each agent adopts the truthful strategy. Consider the case when task 1 is the bonus task. Half the time CA assigns task 2 as the penalty task for the first agent and task 3 as the penalty task for the second agent. Otherwise, the roles of the penalty tasks are reversed. Suppose task 1 is used as the bonus task, task 2 and 3 as the penalty tasks respectively for the first and the second agent. Then the expected score is

$$\begin{aligned} \sum_{i,j} P_1(i, j)S_1(i, j) - P_2(i)P_3(j)S_1(i, j) &= \sum_{i,j: \Delta_1(i,j) > 0} P_1(i, j) - P_2(i)P_3(j) \\ &= P_1(Y, Y) - P_2(Y)P_3(Y) + P_1(N, N) - P_2(N)P_3(N) \\ &= 0.4 - 0.84 * 0.62 + 0.16 - 0.16 * 0.38 = -0.0216 \end{aligned}$$

A similar calculation shows that when tasks 3 and 2 are the penalty tasks for the first and second agent respectively, the expected payment is  $-0.0216$ . So, the expected score to either agent when task 1 is the bonus task and both are truthful is  $-0.0216$ . Similarly, we can show that the expected scores are  $-0.1912$  and  $-0.0216$  when tasks 2 and 3 are respectively the bonus tasks.

Now consider the situation when the first agent always reports  $N$ . Suppose task 1 is the bonus task and tasks 2 and 3 are respectively the penalty tasks for the first and second agent. Then the expected score for the first agent is

$$\begin{aligned} \sum_{i,j} P_1(i, j)S_1(N, j) - P_2(i)P_3(j)S_1(N, j) &= \sum_j S_1(N, j) \left\{ \sum_i P_1(i, j) - P_3(j) \sum_i P_2(i) \right\} \\ &= \sum_j S_1(N, j) \{P_1(j) - P_3(j)\} = P_1(N) - P_3(N) = 0 \end{aligned}$$

A similar calculation shows that when task 3 is the penalty task for the first agent and task 2 is the penalty task for the second agent, the expected score is  $P_1(N) - P_2(N) = 0.22$ . So on average, the expected score of the first agent when task 1 is the bonus task is  $0.5 * (0 + 0.22) = 0.11$ . Similarly, we can show that when tasks 2 and 3 are used as bonus tasks, the expected scores to the first agent are respectively 0.22 and 0.11. Therefore, this example illustrates that CA fails to be informed truthful when the tasks are heterogeneous.

### 3 The Correlated-Agreement Heterogeneous (CAH) Mechanism

In this section, we extend the CA mechanism to handle heterogeneous tasks. The main idea is to use a modified delta matrix for each bonus task, this delta matrix capturing the correlation between a pair of signals on  $b$  and a pair of signals on two, distinct penalty tasks. Algorithm 1 describes the CAH mechanism for rewarding responses to a bonus task  $b$ .

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#### Algorithm 1 CAH mechanism

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**Require:** Joint probability distribution  $P_b(\cdot, \cdot)$ , marginal probability distributions  $\{P_l(\cdot)\}_{l \neq b}$  and reports  $\{r_k^1, r_k^2\}_{k=1}^m$   
 $l' \leftarrow$  uniformly at random from  $\{1, \dots, m\} \setminus \{b\}$  (penalty task assigned to agent 1)  
 $l'' \leftarrow$  uniformly at random from  $\{1, \dots, m\} \setminus \{b, l'\}$  (penalty task assigned to agent 2)

$$\Delta_b(i, j) = P_b(i, j) - \frac{1}{(m-1)(m-2)} \sum_{(t', t'') : t', t'' \in [m] \setminus \{b\}} P_{t'}(i) P_{t''}(j) \quad (4)$$

Let  $S_b(i, j)$  be the corresponding score matrix i.e.

$$S_b(i, j) = 1 \text{ if } \Delta_b(i, j) > 0 \text{ and } S_b(i, j) = 0 \text{ otherwise}$$

Make payment  $S_b(r_b^1, r_b^2) - S_b(r_{l'}^1, r_{l''}^2)$  to each agent

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#### 3.1 Analysis of the CAH Mechanism

First we note that it is sufficient to consider only deterministic strategies. The proof of this statement is analogous to Lemma 3.2 [Shnayder et al., 2016], and uses the fact that the maximization of a linear function over a convex region is extremal. Let  $F_i$  ( $G_j$ ) denote the report of agents 1 (2) on signal  $i$  ( $j$ ). Let  $l'$  and  $l''$  to denote the penalty tasks assigned to agents 1 and 2 respectively. The expected score for strategies  $F$  and  $G$ , conditioned on bonus task  $b$ , is:

$$E_b(F, G) = \mathbf{E}_{l', l''} \left[ \sum_{i, j} P_b(i, j) S_b(F_i, G_j) - \sum_{i, j} P_{l'}(i) P_{l''}(j) \right] \quad (5)$$

$$= \sum_{i, j} P_b(i, j) S_b(F_i, G_j) - \sum_{(l', l'') \in [m] \setminus \{b\}} \frac{1}{(m-1)(m-2)} \sum_{i, j} P_{l'}(i) P_{l''}(j) S_b(F_i, G_j) \quad (6)$$

$$= \sum_{i, j} \Delta_b(i, j) S_b(F_i, G_j) \quad (7)$$

Therefore, the expected score, averaged over the  $m$  possible bonus tasks, is

$$E(F, G) = \frac{1}{m} \sum_{b=1}^m E_b(F, G) = \frac{1}{m} \sum_{b=1}^m \sum_{i, j} \Delta_b(i, j) S_b(F_i, G_j) \quad (8)$$

We now state a property about the delta matrices (4).

**Lemma 1.** For each task  $b$ , we have  $\sum_{i, j} \Delta_b(i, j) = 0$

*Proof.*

$$\begin{aligned}\sum_i \sum_j \Delta_b(i, j) &= \sum_i \sum_j \left\{ P_b(i, j) - \frac{1}{(m-1)(m-2)} \sum_{l' \in [m] \setminus \{b\}} \sum_{l'' \in [m] \setminus \{b, l'\}} P_{l'}(i) P_{l''}(j) \right\} \\ &= \sum_i \left\{ P_b(i) - \frac{1}{(m-1)(m-2)} \sum_{l' \in [m] \setminus \{b\}} \sum_{l'' \in [m] \setminus \{b, l'\}} P_{l'}(i) \right\} = 1 - 1 = 0\end{aligned}$$

□

### 3.2 Strong Truthfulness

We state a sufficient condition for the CAH mechanism to satisfy the property of strong truthfulness.

**Condition 1 :**

1.  $\Delta_b(i, i) > 0, \quad \forall b \forall i.$
2.  $\sum_{b=1}^m \Delta_b(i, j) < 0, \quad \forall i \neq j.$

**Theorem 2.** *If  $\{\Delta_b\}_{b=1}^m$  satisfy Condition 1, then the CAH mechanism is strongly truthful.*

*Proof.* See Appendix. □

Condition 1 is slightly different from the *categorical* condition [Shnayder et al., 2016].  $\Delta_b$  is categorical if (1)  $\Delta_b(i, i) > 0$  for all signals  $i$ , and (2)  $\Delta_b(i, j) < 0$  whenever  $i \neq j$ . Condition 1 does not require every off-diagonal entry to be negative for all tasks  $b$ , but only that the average of the off-diagonal entries is negative.

**Theorem 3.** *Condition 1 and the categorical condition are equivalent when there are only two signals.*

*Proof.* Suppose there three tasks  $(b, l'$  and  $l'')$  with two signals ( $Y$  and  $N$ ) for each task. Matrix  $\Delta_b$  is:

	$Y$	$N$
$Y$	$P_b(Y, Y) - P_{l'}(Y)P_{l''}(Y)$	$P_b(Y, N) - \frac{1}{2}(P_{l'}(Y)P_{l''}(N) + P_{l''}(Y)P_{l'}(N))$
$N$	$P_b(N, Y) - \frac{1}{2}(P_{l'}(N)P_{l''}(Y) + P_{l''}(N)P_{l'}(Y))$	$P_b(N, N) - P_{l'}(N)P_{l''}(N)$

Since the agents are exchangeable,  $P_b$  and  $\Delta_b$  are symmetric. Moreover, Lemma 1 proves that the sum of all the entries of  $\Delta_b$  is zero. This implies that  $\Delta_b(Y, Y) + \Delta_b(N, N) = -2\Delta_b(Y, N)$  and  $\Delta_b(Y, Y) > 0$  and  $\Delta_b(N, N) > 0 \Rightarrow \Delta_b(Y, N) < 0$ . Therefore,  $\Delta_b$  is categorical for all  $b$  iff  $\{\Delta_b\}_{b=1}^m$  satisfy Condition 1. □

### 3.3 Informed Truthfulness

We now establish that the CAH mechanism is informed truthful under a very weak condition on the signal distributions for each task.

**Theorem 4.** *If for each task  $b$ ,  $\Delta_b$  is symmetric and each entry of  $\Delta_b$  is non-zero, then the CAH mechanism is informed truthful.*

*Proof.* For any bonus task  $b$ , the truthful strategy  $(\mathbb{I}, \mathbb{I})$  has higher expected score than any other pair of strategies  $F, G$ :

$$E_b(\mathbb{I}, \mathbb{I}) = \sum_{i, j} \Delta_b(i, j) S_b(i, j) \quad (9)$$

$$= \sum_i \sum_j \max(0, \Delta_b(i, j)) \geq \sum_{i, j} \Delta_b(i, j) S_b(F_i, G_j) = E(F, G). \quad (10)$$

Consider an uninformed strategy  $F$ , with  $F_i = r$  for all  $i$ . Then for any  $G$ , the expected score is

$$\sum_{i=1}^n \sum_{j=1}^n \Delta_b(i, j) S_b(r, G_j) = \sum_{j=1}^n S_b(r, G_j) \sum_{i=1}^n \Delta_b(i, j) \leq \sum_{j=1}^n \max(0, \sum_{i=1}^n \Delta_b(i, j)). \quad (11)$$

We need to show the following:

$$\sum_{j=1}^n \max(0, \sum_{i=1}^n \Delta_b(i, j)) < \sum_{j=1}^n \sum_{i=1}^n \max(0, \Delta_b(i, j)). \quad (12)$$

It is enough to show that for each  $b$ , there exists a column  $j$  and two different rows  $i_1, i_2$  such that

$$\Delta_b(i_1, j) > 0 \text{ and } \Delta_b(i_2, j) < 0. \quad (13)$$

Suppose not. Then each column of  $\Delta_b$  has either all positive entries or all negative entries. Since each entry of  $\Delta_b$  is non-zero and Lemma 1 holds, there exist two columns  $j_1$  and  $j_2$  such that all entries of  $j_1$  ( $j_2$ ) are positive (negative). But, this implies  $\Delta_b(j_2, j_1) > 0$  and  $\Delta_b(j_1, j_2) < 0$ , which contradicts the fact that  $\Delta_b$  is symmetric.  $\square$

## 4 Combining CAH with Estimation

As with the CA mechanism [Shnayder et al., 2016], the CAH mechanism remains (approximately) informed truthful even when the statistics used to determine scores are estimated from the reports of strategic agents. The reason for this robustness is that the score matrix that corresponds to the correct statistics is the best possible score matrix for agents (even if they could cooperate in designing an alternate matrix.)

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### Algorithm 2 Detail-Free CAH mechanism

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**Require:** Agent  $p$  of a population of  $q$  agents provides reviews  $(r_1^p, \dots, r_m^p)$  on each of the  $m$  tasks.  
 $T_k(i, j) \leftarrow$  observed frequency of signal pair  $i, j$  on task  $k$ .  
Pair up the agents uniformly at random and then run CAH for each pair with the estimated distribution  $\{T_k(\cdot, \cdot)\}_{k=1}^m$

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The detail-Free CAH mechanism is  $(\varepsilon, \delta)$ -informed truthful:

**Theorem 5.** *If there are at least  $q \geq \frac{9n}{\varepsilon^2} \log\left(\frac{m}{\delta}\right)$  agents reviewing each task, for  $m$  tasks and  $n$  possible signals, then with probability at least  $1 - \delta$ ,*

$$E[\mathbb{I}, \mathbb{I}] \geq E[F, G] - \varepsilon \quad \forall F, G$$

*Proof.* See Appendix.  $\square$

We leave to future work an analogous statement for a setting where tasks are from a number of discrete distribution types.

## 5 Experimental Results

In this section we compare three different peer prediction mechanisms on heterogeneous tasks: the CAH mechanism, and two non multi-task mechanisms, namely the *robust peer truth serum* (RPTS) mechanism [Radanovic et al., 2016] (which sets a score of  $1/P(i)$  for agreement on signal  $i$ ) and the *Kamble* mechanism [Kamble et al., 2015] (which sets a score for agreement on signal  $i$  of  $1/\sqrt{P(i, i)}$ ).

We evaluate these mechanisms on an anonymized dataset from Google Local Guides, which is a platform for collecting user generated content in regard to places on Google Maps. A user can provide information by answering ‘yes’, ‘no,’ or ‘not sure’ to a series of questions.<sup>2</sup> Currently, a user is

<sup>2</sup>We decided to ignore the ‘not sure’ response for a question in this initial study, because the semantics is not clear: does this mean the user has missing information, or does it mean the question is not relevant to the location. Thus, it is not *a priori* clear whether to expect correlation between different reports.

awarded points on Google Local Guides based on her quantity of contribution, where a contribution can be a review or a photograph of a place as well as any update about the place of Google maps. Google awards one point for each contribution with a maximum of five points per place. Based on the number of points received a user can be in any one of five levels on the platform, with higher levels providing better benefits such as free Google Drive space, visibility on the Local Guides channel, and access to Google products before they are released for the public.

We estimate pairwise signal distributions from data that represents user reports for Google Local Guides in the US between **June 2015** and **May 2016**. Each task is an instance of a particular type of task, with a type of task specified by a triple from

$$Region \times BusinessType \times Question$$

A region is a US state, there are four business types (these are anonymized in our data, but could be “restaurant,” “bar,” or “public location”), and there are 143 distinct questions across all the regions and business types. The questions are anonymized in our data, but were first categorized by Google as being “subjective” or “factual” questions (e.g., “is this restaurant noisy?” vs “does this cafe have free wifi?”). A (region, business type, question) corresponds to a distinct type of task, and has a corresponding pairwise signal distribution.

The data are counts of pairs of signal reports, broken down by (region, business type, question). As a first step, we estimate the relevant statistics from the data, and then test the mechanisms for different kinds of strategic behaviors.<sup>3</sup> For example, for a type of task  $k$  corresponding to a particular (region, business type, question) we let  $\hat{P}_k(i, j)$  denote the frequency of pairs of responses  $i$  and  $j$  and  $\hat{P}_k(i) = \sum_j \hat{P}_k(i, j)$  denote the marginal distribution. These estimates are used to define scores in each mechanism. The number of different questions (and thus types of tasks) per pair of region and business type varies from 75 to 135, with an average of 102. There were 51 regions (states) and 4 business types per state. The total number of (region, business type, question) triples about which we have data is 20,885.

We will present some analysis at the level of (region, business type) pairs. For CAH, we first compute the delta matrices for each task type using eq. (4). For this, we assume for a given (region, business type, question) that the penalty tasks are sampled from other questions associated with this (region, business type). From these delta matrices, we then use eq. (7) to compute the expected score for each question and then average these scores over all questions associated with a (region, business type) pair. We take a similar approach for computing the average expected score for a (region, business type) under RPTS and Kamble, where we average over the expected score for running the peer prediction mechanism separately on each different question associated with the (region, business type) pair. Finally, since the payments of CAH are bounded between 0 and 1, we normalize the payments of RPTS and Kamble to  $[0, 1]$ .<sup>4</sup>

## 5.1 Resistance to Unilateral Deviations

We consider three kinds of strategic behaviors: *constant-0* (report ‘yes’ all the time), *constant-1* (report ‘no’ all the time) and *random* (report ‘yes’ w.p. 0.5). Figure 1 shows the expected benefit to being truthful vs playing each one of these strategies, considering the three different mechanisms and the effect on average score for each (region, business type). We consider the benefit from being truthful vs adopting some other behavior for three different assumptions about the rest of the population: we vary the fraction  $p$  (0, 0.5 or 1) of the population that are truthful, with the remaining  $1 - p$  fraction using the same strategic behavior as the agent. For example, the  $p = 0.5$  histograms show the distribution of the advantage of truthful behavior relative to each possible strategic behavior, when half of the population follows the same strategic behavior. As more of the population is truthful, the truthful behavior becomes increasingly beneficial. In particular, looking at the plots for  $p = 1$  we see all three mechanisms provide positive incentives for truthful vs strategic behavior. However, when  $p$  is 0 or 0.5 and other agents are also strategic, the CAH mechanism retains an incentive for

<sup>3</sup>We do not re-estimate statistics based on these simulated strategic behaviors.

<sup>4</sup>Suppose there are two signals 0 and 1, with prior probabilities  $P(0) > P(1)$ . Then RPTS would pay more on signal 1 than signal 0. Because of this, we set the payment to 1 for agreement on signal 1, and effectively divide all payments through by the unnormalized payment for agreement on signal 1 (i.e.,  $1/P(1)$ .) The rewards of Kamble are normalized analogously.

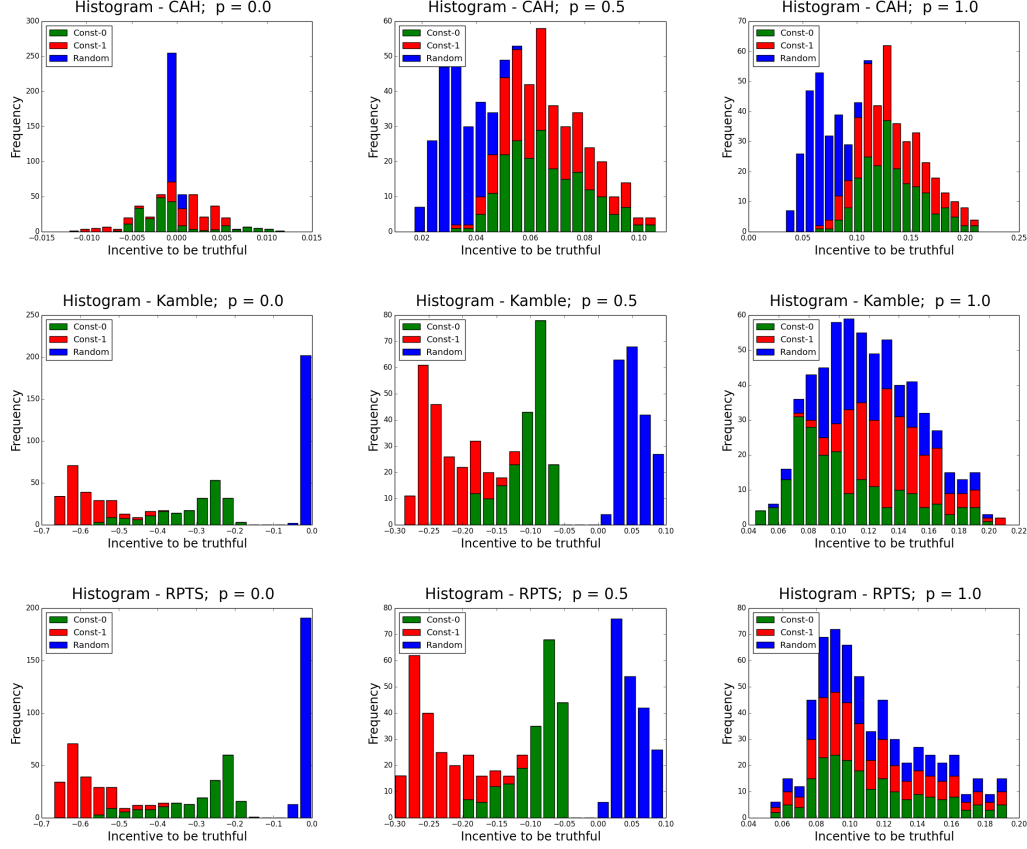


Figure 1: Histograms for the 204 (region, business type) pairs of expected benefit (averaged across questions) from truthful behavior vs. some other strategy, when fraction  $p$  is truthful and  $1 - p$  plays the same strategy. We consider  $p = 0, 0.5$  and  $p = 1$ .

truthful behavior while const-0 and const-1 strategies become beneficial for agents in the RPTS and Kamble mechanisms. This analysis shows that truthful reporting is a best response for an agent on average, over all questions associated with a (region, business type) pair, irrespective of the behavior of the rest of the population. For the particular case of  $p = 0$ , when the other agents all follow some non-truthful strategy, the average score in the CAH mechanism is zero irrespective of the behavior of an agent.

## 5.2 Resistance to Coordinated Misreports

Figure 2 compares the robustness of each mechanism to coordinated misreports, looking at the average score for questions associated with a particular (region, business type) pair. The results are qualitatively similar for other (region, business type) pairs. For each strategy, we plot the average score when  $p$  fraction of population is truthful and the remaining  $1 - p$  fraction of the population adopts the same strategy, and vary  $p$ . Irrespective of the mechanism, we again see that the expected score is higher from truthful behavior than the other strategies when the rest of the population is truthful. We also see that the average score from all-truthful is better than other behaviors in the CAH mechanism, even when all the population coordinates on a particular behavior. In contrast, both Kamble and RPTS are prone to coordinated deviation to either constant-1 or constant-0.<sup>5</sup>

<sup>5</sup>This is without reestimating the statistics that are induced by these misreports. We can think about this as a successful, short-term population-level strategy.



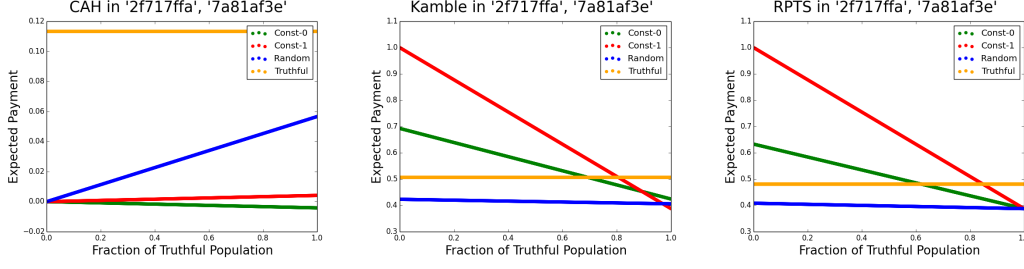


Figure 2: Expected score for following each of four strategies, when  $p$  fraction of the population is truthful and  $1 - p$  fraction adopt the same strategy. Averaged over questions associated with a typical (region, business type) pair.

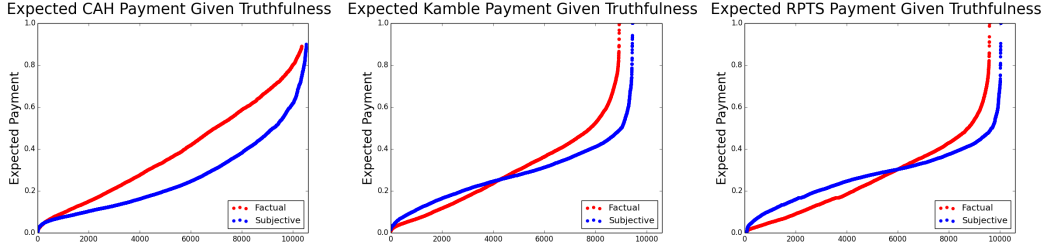


Figure 3: Cumulative distribution on expected payments at truthful reporting in each mechanism, with results separated into questions that are categorized as ‘factual’ and those that are categorized as ‘subjective.’

### 5.3 Subjective vs Factual Tasks

Figure 3 shows the cumulative distribution on expected scores at truthful reporting in each mechanism, where each data point corresponds to a different (region, business type, question) triple. Two lines are shown for each mechanism: one corresponding to questions that are categorized as ‘factual’ and one corresponding to questions that are categorized as ‘subjective.’ The subjective questions tend to provide lower expected payment than the factual questions under the CAH mechanism. This is consistent with the intuition that people perceive subjective questions differently than factual questions. For the Kamble and RPTS mechanisms, the variability in expected payment is larger across factual questions than subjective questions, with the expected payment for subjective questions tending to fall in a narrow band.

## 6 Conclusions

We study the peer prediction problem when users complete heterogeneous tasks. We introduce the CAH mechanism, which is informed-truthful under mild conditions and can also be used together with estimating statistics from reports for the purpose of computing scores. The experimental results suggest that CAH provides better incentive for being truthful and is more resistant to coordinated misreports than the RPTS and Kamble mechanisms. In future work we should also compare the mechanisms when we re-estimate statistics under misreports. One additional consideration is that the reports from a user in one session may not be independent (e.g., responses to questions about the same place in the context of Google Local Guides.) Finally, it is important in future work to handle agent heterogeneity as well as task heterogeneity.

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## A Proof of theorem 2

Suppose both the agents adopt the truthful strategy, which corresponds to the identity matrix  $\mathbb{I}$ . Then the expected payment is given as

$$E(\mathbb{I}, \mathbb{I}) = \sum_{b=1}^m \sum_{i,j: \Delta_b(i,j) > 0} \Delta_b(i,j) \quad (14)$$

On the other hand for any two arbitrary deterministic strategies  $F$  and  $G$ ,

$$E(F, G) = \sum_{b=1}^m \sum_{i,j} \Delta_b(i,j) S_b(F_i, G_j) \leq \sum_{b=1}^m \sum_{i,j: \Delta_b(i,j) > 0} \Delta_b(i,j) = E(\mathbb{I}, \mathbb{I}) \quad (15)$$

To show strong truthfulness, consider an asymmetric joint strategy  $F \neq G$ . Then there exists  $i$  such that  $F_i \neq G_i$ . This reduces the expected payment by at least

$$\sum_{b=1}^m \Delta_b(i, i) S_b(F_i, G_i) \quad (16)$$

Since  $F_i \neq G_i$ , we have  $\sum_{b=1}^m \Delta_b(F_i, G_i) < 0$  and there exists  $l'$  such that  $\Delta_{l'}(F_i, G_i) < 0$  (or  $S_{l'}(F_i, G_i) = 0$ ). Therefore, the expected payment reduces by at least  $\Delta_{l'}(i, i) > 0$ .

Now consider symmetric, non-permutation strategy  $F = G$ . Then there exist  $i \neq j$  such that  $F_i = G_j = k$  and the expected payment includes

$$\sum_{b=1}^m \Delta_b(i, j) S_b(k, k) = \sum_{b=1}^m \Delta_b(i, j) < 0 \quad (17)$$

The first equality uses the fact  $S_b(k, k) = 1$  since  $\Delta_b(k, k) > 0$  for each  $b$ .

## B Proof of theorem 5

We will write  $E[T, F, G]$  to denote the average expected score under strategies  $F$  and  $G$  when using the score matrix  $T = \{T_b\}_{b=1}^m$ . Suppose  $S = \{S_b\}_{b=1}^m$  is the true scoring matrix and  $\hat{S} = \{\hat{S}_b\}_{b=1}^m$  is the scoring matrix estimated from the data. Then

$$E[\hat{S}, F, G] = \frac{1}{m} \sum_{b=1}^m \sum_{i,j} \Delta_b(i, j) \hat{S}_b(F_i, G_j) \leq \frac{1}{m} \sum_{b=1}^m \sum_{i,j: \Delta_b(i,j) > 0} \Delta_b(i, j) = E[S, \mathbb{I}, \mathbb{I}] \quad (18)$$

Therefore, in order to show  $E[\hat{S}, \mathbb{I}, \mathbb{I}] \geq E[\hat{S}, F, G] - \varepsilon$  it is enough to show that  $E[\hat{S}, \mathbb{I}, \mathbb{I}] \geq E[S, \mathbb{I}, \mathbb{I}] - \varepsilon$ . Now

$$\begin{aligned} & \left| \frac{1}{m} E[\hat{S}, \mathbb{I}, \mathbb{I}] - \frac{1}{m} E[S, \mathbb{I}, \mathbb{I}] \right| \\ &= \left| \frac{1}{m} \sum_{b=1}^m \sum_{i,j} \Delta_b(i, j) (\hat{S}_b(i, j) - S_b(i, j)) \right| = \left| \frac{1}{m} \sum_{b=1}^m \sum_{i,j} \Delta_b(i, j) (\text{sign}(\hat{\Delta}_b(i, j)) - \text{sign}(\Delta_b(i, j))) \right| \\ &\leq \frac{1}{m} \sum_{b=1}^m \sum_{i,j} |\Delta_b(i, j) (\text{sign}(\hat{\Delta}_b(i, j)) - \text{sign}(\Delta_b(i, j)))| \leq \frac{1}{m} \sum_{b=1}^m \sum_{i,j} |\hat{\Delta}_b(i, j) - \Delta_b(i, j)| \\ &= \frac{1}{m} \sum_{b=1}^m \sum_{i,j} \left| P_b(i, j) - T_b(i, j) - \frac{1}{(m-1)(m-2)} \sum_{(t', t''): t', t'' \in [m] \setminus \{b\}} (P_{t'}(i) P_{t''}(j) - T_{t'}(i) T_{t''}(j)) \right| \\ &\leq \frac{1}{m} \sum_{b=1}^m \sum_{i,j} |P_b(i, j) - T_b(i, j)| + \frac{1}{(m-1)(m-2)} \sum_{(t', t''): t', t'' \in [m] \setminus \{b\}} |P_{t'}(i) P_{t''}(j) - T_{t'}(i) T_{t''}(j)| \\ &= \frac{1}{m} \sum_{b=1}^m \sum_{i,j} |P_b(i, j) - T_b(i, j)| + \frac{1}{m(m-1)(m-2)} \sum_{b=1}^m \sum_{i,j} \sum_{(t', t''): t', t'' \in [m] \setminus \{b\}} |P_{t'}(i) (P_{t''}(j) - T_{t''}(j)) \\ &\quad + T_{t''}(j) (P_{t'}(i) - T_{t'}(i))| \\ &\leq \frac{1}{m} \sum_{b=1}^m \sum_{i,j} |P_b(i, j) - T_b(i, j)| \end{aligned} \quad (19)$$

$$\begin{aligned} &+ \frac{1}{m(m-1)(m-2)} \sum_{b=1}^m \sum_{(t', t''): t', t'' \in [m] \setminus \{b\}} \left\{ \sum_j |P_{t''}(j) - T_{t''}(j)| \sum_i P_{t'}(i) + \sum_i |P_{t'}(i) - T_{t'}(i)| \sum_j T_{t''}(j) \right\} \\ &= \frac{1}{m} \sum_{b=1}^m \sum_{i,j} |P_b(i, j) - T_b(i, j)| \end{aligned} \quad (20)$$

$$\begin{aligned} &+ \frac{1}{m(m-1)(m-2)} \sum_{b=1}^m \sum_{(t', t''): t', t'' \in [m] \setminus \{b\}} \left\{ \sum_j |P_{t''}(j) - T_{t''}(j)| + \sum_i |P_{t'}(i) - T_{t'}(i)| \right\} \end{aligned} \quad (21)$$

Now if we have  $O\left(\frac{9n^2}{\varepsilon^2} \log\left(\frac{m}{\delta}\right)\right)$  samples from each joint distribution  $P_b$  (where  $n$  is the number of signals) and  $O\left(\frac{9n}{\varepsilon^2} \log\left(\frac{m}{\delta}\right)\right)$  from each marginal distribution  $P_b$ , we can ensure that with probability at least  $1 - \delta$ , for all  $b = 1, 2, \dots, m$  the following results hold (see Devroye and Lugosi [2012] for a proof)

$$\sum_{i,j} |P_b(i, j) - T_b(i, j)| \leq \frac{\varepsilon}{3} \text{ and } \sum_i |P_b(i) - T_b(i)| \leq \frac{\varepsilon}{3}. \quad (22)$$

Note: If we just had  $O(n/\varepsilon^2 \log(1/\delta))$  samples for each task, then we can guarantee (22) for each task separately with probability at least  $1 - \delta$ . By the union bound, this would give a success probability

of  $1 - m\delta$  over all tasks. So in order to have a  $1 - \delta$  confidence bound, we need a  $\log(m/\delta)$  factor in the sample complexity. Substituting the bounds from eq. (22) in eq. (21) and simplifying gives us  $\left| E \left[ \hat{S}, \mathbb{I}, \mathbb{I} \right] - E \left[ S, \mathbb{I}, \mathbb{I} \right] \right| \leq \varepsilon$ . Since there are  $q$  agents providing reviews for each task, we get  $q^2$  samples from each joint distribution and  $q$  samples from each marginal distribution. So as long as  $q \geq \frac{9n}{\varepsilon^2} \log \left( \frac{m}{\delta} \right)$  we have enough number of samples and we are done.